A *Y*-Junction Strip-Line Circulator*

U. MILANO†, J. H. SAUNDERS†, AND L. DAVIS, JR.†

Summary—The theoretical approach to the three-port symmetrical circulator is reviewed and presented in a form valid for the most general waveguide case.

A strip-line *Y*-junction circulator is described and the performance of different units in the band 800–1600 mc is illustrated.

The new type of device described offers, for the low-frequency region of the microwave spectrum, advantages of simple design, light weight, and great compactness with respect to the classical types. When operated with a permanent magnet it gives—in a bandwidth of about 4 per cent—isolation greater than 20 db, insertion loss ≤ 0.4 db, and input VSWR ≤ 1.20.

I. Introduction

RECENTLY, several people have worked on microwave circulators consisting of symmetrical waveguide junctions containing symmetrically located pieces of ferrite.[1–5]

The experimental results which have been published concern, in general, turnstile junctions or n-port star junctions of rectangular waveguides; the present paper deals with a TEM structure realized in strip-line which can offer distinct advantages of compactness, small weight, and structural simplicity when applications in the low-frequency region of the microwave spectrum are considered.

First of all, some theoretical considerations are reviewed which can illustrate the behavior of this type of circulator, and afterwards some experimental results obtained in the range 800–1600 mc are presented.

II. The Mathematical Approach to the Three-Port Symmetrical Circulator

The scattering matrix formalism and the application of group theory have been proven to be very useful for approaching the theoretical problems of symmetrical waveguide circulators.

Treuhaft[6] has shown the correspondence between the scattering matrix of circulators and the cyclic substitutions of the group theory, and he has investigated on this basis some general symmetry properties of the circulators. Auld[4] has applied these mathematical tools to the general problem of the synthesis of symmetrical waveguide circulators; Sirvetz[7] has applied the scattering matrix formalism for a perturbation treatment of the Y-junction circulator.

One of the interesting features of the scattering matrix approach is that it can be applied to the most general type of waveguide, including in the term "waveguide" also the TEM structures. Therefore, we will follow this approach and we will consider in this section the most general case of a lossless symmetrical Y-junction of three microwave lines containing at its center a ferrite medium located in such a way that it does not alter the symmetry of the junction (Fig. 1) and magnetized along the axis of the junction.

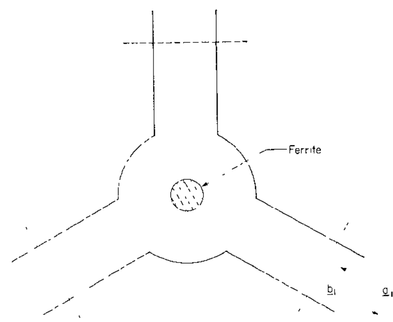

Fig. 1—Microwave Y-junction.

In order that this junction be a circulator—according to Treuhaft's definition[6]—its scattering matrix must operate on the incident waves so as to produce the same result as a cyclic substitution.[8] That is to say, if we indicate with E_1^i, E_2^i, E_3^i, the transverse components of the incident electric fields and with E_1^r, E_2^r, E_3^r, the transverse components of the reflected electric fields at the reference planes of the junction, the scattering matrix must produce the same result as the substitution

$$[E_1^r\, E_2^r\, E_3^r] = \{(1\ 2\ 3) \rightarrow [E_1^i\, E_2^i\, E_3^i]\} = [E_2^i\, E_3^i\, E_1^i] \qquad (1)$$

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[1] P. J. Allen, "The turnstile circulator," IRE Trans. on Microwave Theory and Techniques, vol. MTT-4, pp. 223–227; October, 1956.

[2] T. Schaug-Pettersen, ONR London Tech. Rept. ONRL 111-57; September, 1957.

[3] H. N. Chait and T. E. Curry, "*Y* circulator," *J. Appl. Phys.*, suppl. to vol. 30, pp. 152S–153S; April, 1959.

[4] B. A. Auld, "The synthesis of symmetrical waveguide circulators," IRE Trans. on Microwave Theory and Techniques, vol. MTT-7, pp. 238–246; April, 1959.

[5] S. Yoshida, "*X* circulator," Proc. IRE, vol. 47, p. 1150; June, 1959.

[6] M. A. Treuhaft, "Network properties of circulators based on the scattering concept," Proc. IRE, vol. 44, pp. 1394–1402; October, 1956.

[7] M. H. Sirvetz, "The *Y*-Junction Microwave Circulator," Raytheon Co, Waltham, Mass., Tech. Memo T-143; March, 1959.

[8] By the cyclic substitution (abc) operating on a sequence of three elements, we mean an operation that substitutes the element a by b, b by c, and c by a. If, for example, we apply such a substitution to the sequence bac and we indicate it with the symbol $\{(abc)\rightarrow bac\}$, we obtain the result:

$$\{(abc) \rightarrow bac\} = cba.$$

for one sense of circulation, or the substitution

$$[E_1^r\, E_2^r\, E_3^r] = \{(1\ 3\ 2) \rightarrow [E_1^i\, E_2^i\, E_3^i]\} = [E_3^i\, E_1^i\, E_2^i] \tag{2}$$

for the opposite sense of circulation.

These substitutions must correspond, using the matrix notation, to

$$\begin{bmatrix} E_1^r \\ E_2^r \\ E_3^r \end{bmatrix} = \boldsymbol{S}^{(c)} \begin{bmatrix} E_1^i \\ E_2^i \\ E_3^i \end{bmatrix} \tag{3}$$

where $\boldsymbol{S}^{(c)}$ indicates the scattering matrix of the three-port circulator. We will restrict ourselves to consider only the sense of circulation corresponding to the substitution (1), and one immediately sees that the scattering matrix required to perform this operation is

$$\boldsymbol{S}^{(c)} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}. \tag{4}$$

The form (4) assumes a suitable choice of the reference planes to correspond to a zero phase shift through the junction.

We now represent with the complex number a_i ($i=1, 2, 3$) the amplitude and phase of the transverse electric field of the incident wave at the ith reference plane, normalized in such a way that the average incident power is given by $\frac{1}{2}a_i^*a_i$; we also indicate with b_i the corresponding measure of the emergent wave. If we indicate with $\boldsymbol{a}$ and $\boldsymbol{b}$ the column vectors

$$\boldsymbol{a} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}; \quad \boldsymbol{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix},$$

we can write (3) in the form

$$\boldsymbol{b} = \boldsymbol{S}^{(c)}\boldsymbol{a}. \tag{3$'$}$$

Let us consider the eigenvalue equation of the square matrix $\boldsymbol{S}^{(c)}$:

$$\boldsymbol{S}^{(c)}\boldsymbol{a}^{(c)} = s^{(c)}\boldsymbol{a}^{(c)} \tag{5}$$

where $\boldsymbol{a}^{(c)}$ is now an eigenvector and $s^{(c)}$ an eigenvalue.

Eq. (5) has a nonvanishing solution for $\boldsymbol{a}^{(c)}$ if the following condition is satisfied:

$$\det \left| \boldsymbol{S}^{(c)} - s^{(c)}\boldsymbol{I} \right| = 0 \tag{6}$$

where $\boldsymbol{I}$ indicates the unity matrix.

The solution of the characteristic equation obtained by (6) gives the following set of nondegenerate eigenvalues:

$$s_1^{(c)} = 1$$
$$s_2^{(c)} = e^{j(2\pi/3)}$$
$$s_3^{(c)} = e^{j(2\pi/3)}. \tag{7}$$

By substituting these values in (5), one obtains for the normalized eigenvectors the set

$$\boldsymbol{a}^{(1)} = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}; \quad \boldsymbol{a}^{(2)} = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 \\ e^{-j(2\pi/3)} \\ e^{j(2\pi/3)} \end{bmatrix};$$
$$\boldsymbol{a}^{(3)} = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 \\ e^{j(2\pi/3)} \\ e^{-j(2\pi/3)} \end{bmatrix}. \tag{8}$$

It is rather interesting to compare these results with those one obtains in the case of the reciprocal Y-junction. If we indicate with $\boldsymbol{S}^{(r)}$ the matrix of the reciprocal junction and with $S_{ij}^{(r)}$ the generic element of the matrix, we realize, by inspecting the symmetry properties of the junction, that the following equalities must hold:

$$S_{11}^{(r)} = S_{22}^{(r)} = S_{33}^{(r)} = S'$$
$$S_{12}^{(r)} = S_{21}^{(r)} = S_{13}^{(r)} = S_{31}^{(r)} = S_{23}^{(r)} = S_{32}^{(r)} = S''. \tag{9}$$

That is to say,

$$\boldsymbol{S}^{(r)} = \begin{bmatrix} S' & S'' & S'' \\ S'' & S' & S'' \\ S'' & S'' & S' \end{bmatrix}. \tag{10}$$

By solving the determinental equation

$$\det \left| \boldsymbol{S}^{(r)} - s^{(r)}\boldsymbol{I} \right| = 0, \tag{11}$$

one finds for the eigenvalues of $\boldsymbol{S}^{(r)}$

$$s_1^{(r)} = S' + 2S''$$
$$s_2^{(r)} = s_3^{(r)} = S' - S''. \tag{12}$$

The two-fold degeneracy of the eigenvalues $s_2^{(r)}$ and $s_3^{(r)}$ leads to an ambiguity in the definition of the eigenvectors $\boldsymbol{a}_2^{(r)}$ and $\boldsymbol{a}_3^{(r)}$; for our purposes, however, it is interesting to notice that one possible set of eigenvectors of $\boldsymbol{S}^{(r)}$, corresponding to the above indicated eigenvalues, is given by the set (8), as one can easily verify.

The eigensolutions corresponding to the set of eigenvectors (8) have been described in the literature.[9] For the eigensolution corresponding to the eigenvector $\boldsymbol{a}^{(1)}$, the electromagnetic field at the center of the junction has only components parallel to the axis of the junction. For the eigensolutions corresponding to the eigenvectors $\boldsymbol{a}^{(2)}$ and $\boldsymbol{a}^{(3)}$, the axial fields vanish at the center of the junction, but the transverse components of the electric and magnetic field give rise there to circularly-polarized waves rotating in one sense for the eigenvector $\boldsymbol{a}^{(2)}$ and in the opposite sense for the eigenvector $\boldsymbol{a}^{(3)}$.

When we introduce into the reciprocal Y-junction a symmetrically-located piece of ferrite, magnetized along the axis of the junction, the relationship $S_{ik} = S_{ki}$ no

[9] C. G. Montgomery, R. H. Dicke, and E. M. Purcell, "Principles of Microwave Circuits," McGraw-Hill Book Co., Inc., New York, N. Y., chs. 5 and 12; 1948.

longer holds, and we obtain a scattering matrix $\mathbf{S}^{(f)}$ that assumes the general form

$$\mathbf{S}^{(f)} = \begin{bmatrix} S' & S'' & S''' \\ S''' & S' & S'' \\ S'' & S''' & S' \end{bmatrix}. \qquad (13)$$

The eigenvector (8) of $\mathbf{S}^{(r)}$ are, however, still eigenvectors of $\mathbf{S}^{(f)}$ because the ferrite does not alter the symmetry of the junction either from the geometrical point of view or from the electromagnetic point of view.

In the device considered here, the ferrite sample, whose transverse dimensions we suppose small in comparison with the wavelength, is located in a region where the RF magnetic field is either directed along the axis of the junction for the first eigensolution, or is circularly-polarized for the other two eigensolutions. In each case, the ferrite presents an effective scalar permeability which affects the corresponding eigenvalues. In particular, $s_2^{(f)}$ and $s_3^{(f)}$ will assume different values because of the different effective permeabilities pertaining to the two corresponding eigensolutions. In other words, the introduction of the ferrite has the effect of removing what has been called the "reciprocity degeneracy" of the Y-junction.[10]

We now observe that $\mathbf{S}^{(f)}$ can be diagonalized by a matrix $\boldsymbol{X}$ having for its columns the eigenvectors of $\mathbf{S}^{(f)}$, *i.e.*, the set (8).[11]

By this we mean that for such a matrix $\boldsymbol{X}$, the following relationship holds:

$$\boldsymbol{X}^{-1}\mathbf{S}^{(f)}\boldsymbol{X} = \boldsymbol{\Lambda} \qquad (14)$$

where

$$\boldsymbol{\Lambda} = \begin{bmatrix} s_1^{(f)} & 0 & 0 \\ 0 & s_2^{(f)} & 0 \\ 0 & 0 & s_3^{(f)} \end{bmatrix}. \qquad (15)$$

In a different form, (14) can be written

$$\mathbf{S}^{(f)} = \boldsymbol{X}\boldsymbol{\Lambda}\boldsymbol{X}^{-1}; \qquad (14')$$

and by multiplying out the right side of (14′), we obtain

$$S_{11}^{(f)} = S_{22}^{(f)} = S_{33}^{(f)} = \frac{s_1^{(f)} + s_2^{(f)} + s_3^{(f)}}{3} = S' \qquad (16)$$

$$S_{12}^{(f)} = S_{23}^{(f)} = S_{31}^{(f)} = \frac{s_1^{(f)} + s_2^{(f)}e^{j(2\pi/3)} + s_3^{(f)}e^{-j(2\pi/3)}}{3} = S'' \qquad (17)$$

$$S_{13}^{(f)} = S_{32}^{(f)} = S_{21}^{(f)} = \frac{s_1^{(f)} + s_2^{(f)}e^{-j(2\pi/3)} + s_3^{(f)}e^{j(2\pi/3)}}{3} = S'''. \qquad (18)$$

In order that the junction of Fig. 1—whose matrix we have called $\mathbf{S}^{(f)}$—be a circulator, it is necessary that the eigenvalues $s^{(f)}$ coincide with the eigenvalues $s^{(c)}$ of $\mathbf{S}^{(c)}$; that is to say, if we represent them on a complex plane, the extremes of the vectors representing them must lie on a circle of unit radius and they must be equally spaced at angles of 120°.

In this situation, the right side of (16) becomes zero, which simply says that, in order that the junction of Fig. 1 be a circulator, it must appear matched at any port. This fact is included in the circulator definition we have assumed, but it is worthwhile to notice, from a practical point of view, that the condition $S'=0$ not only is necessary for the circulator action of a lossless Y-junction, but it is also sufficient, as one can easily see by imposing the unitary condition on the matrix $\mathbf{S}^{(f)}$, together with $S'=0$.[12] Therefore, one can utilize this property for the experimental adjustment of the circulator as we will see in the next section.

III. The Experimental Adjustment of the Y-Junction Strip-Line Circulator

The previous discussion has essentially shown that, in order to obtain a circulator action from the junction schematically illustrated in Fig. 1, the nonreciprocal action of the ferrite has to be such as to remove the degeneracy of the eigenvalues s_2 and s_3 of $\mathbf{S}^{(r)}$ and to obtain the coincidence of the new set of eigenvalues $s_1^{(f)}$, $s_2^{(f)}$, $s_3^{(f)}$ with the set (7) of the eigenvalues $s_i^{(c)}$.

As the vectors representing the $s^{(f)}$ can always be rotated through an arbitrary angle by properly choosing the reference planes of the junction, the above conditions can be expressed as follows:

$$s_2^{(f)} = s_1^{(f)}e^{-j(2\pi/3)} \qquad (19)$$

$$s_3^{(f)} = s_1^{(f)}e^{j(2\pi/3)}. \qquad (20)$$

This shows that the experimental adjustment of the circulator requires the adjustment of two phase angles and therefore the variation of two physical parameters.

In our case, the geometrical configuration of the circulator was realized according to the general structure of Fig. 2. Two disks of ferrite are placed at the center of the junction and they are magnetized along the direction perpendicular to their plane. All the results shown in the next figures refer to the case of disks of polycrystalline YIG polarized above resonance.

The adjustment of the circulator at a fixed frequency was generally done by using as variable parameters the diameter d of the disks and the value H_0 of the polarizing field. These two variable parameters were optimized until the junction appeared to be matched or nearly matched. In this condition, the circulator action is automatically insured.

For any fixed frequency, there is one optimum value of the diameter of the disks which achieves the matched

[10] D. M. Kerns, "Analysis of symmetrical waveguide junctions," *J. Res. NBS*, vol. 46, pp. 267–282; April, 1951.

[11] H. Margenau and G. M. Murphy, "The Mathematics of Physics and Chemistry," D. Van Nostrand Co., New York, N. Y., ch. 10, sec. 15; 1943.

[12] H. J. Carlin, "Principles of gyrator networks," *Proc. Symp. on Modern Advances in Microwave Techniques*, Polytechnic Inst. of Brooklyn, Brooklyn, N. Y., pp. 175–204; November, 1954.

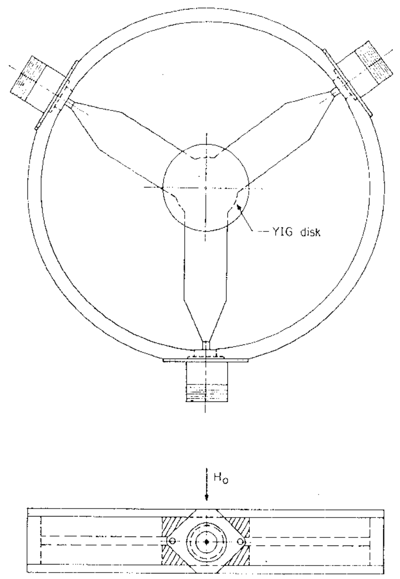

Fig. 2—Y-junction strip-line circulator.

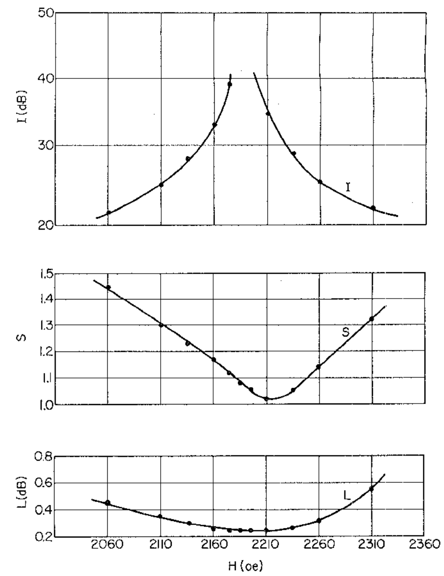

Fig. 3—Performance vs polarizing field at a fixed frequency; $f=1030$ mc.

condition, and this d_{opt} is a continuously decreasing function of the frequency.

By keeping the mechanical dimensions of the junction unchanged and by varying the thickness of the disks, one has another variable parameter that also can be utilized for the adjustment of the circulator. At a fixed frequency and with a fixed mechanical structure, a reduction of the thickness of the disks leads to a greater value of d_{opt}.

Fig. 3 shows some typical results obtained at a fixed frequency by changing the magnetic field when $d \simeq d_{opt}$. We have indicated by I the attenuation between the two decoupled arms ("isolation"), with L the insertion loss between input and forward arm, and with S the VSWR seen at the input arm.

The isolation measurements have always been taken by using a load with an input VSWR of 1.02 on the forward arm and the measurements of the insertion loss have always been taken by using a tunable bolometer.

It appears from the data of Fig. 3 that it is possible, at a single frequency, to associate isolations greater than 30 db with insertion loss of 0.3 db or less, and input VSWR of 1.05.

In Fig. 3, the maximum value of isolation does not correspond to the minimum value of the input VSWR. This is believed to be due to small asymmetries present in the structure.

In Fig. 3 and in the following figures, the values of isolation do not coincide in general with the values one could calculate from the corresponding values of input VSWR in the hypothesis of a lossless circulator.

This may be due to asymmetry effects, but it is shown in the Appendix that the effect of small departures from the optimal conditions for H_0 and d combined with the effect of the losses can also account for this type of deviation.

By choosing the appropriate dimensions of the ferrite and the value of the polarizing magnetic field, it has been possible, by using the mechanical structure indicated in Fig. 2, to cover the range 800–1600 mc with a performance always comparable to that presented in Fig. 3.

The circulator can be operated with a small permanent magnet, and therefore one obtains a simple, very compact device which can give a useful bandwidth of about 4 per cent where the isolation is not less than 20 db, the insertion loss not greater than 0.4 db, and the input VSWR not greater than 1.20.

Figs. 4–6 show some examples of performance obtained in the regions of 800, 1000, and 1300 mc, with some laboratory units polarized with a fixed field.

IV. Conclusion

The mathematical approach to the three-port symmetrical circulator has been reviewed and it has been presented in a form valid for the most general waveguide case. The behavior of the rectangular waveguide three-port symmetrical circulator has been explained by other authors,[3] on the basis of a field displacement effect. The approach illustrated in this paper, which was first used by Auld for the general problem of the synthesis of symmetrical circulators, does not require the hypothesis of a field displacement effect and it seems to the authors more appropriate for the specific experimental type of circulator illustrated here.

The Y-junction strip-line circulator described in the present paper is a new type of device which represents an attractive solution for circulators operating in the low-frequency region of the microwave spectrum where it offers, in comparison with the classical types, distinct advantages of simple design, light weight, and great compactness. This application appears to be of particular interest for use in connection with masers and parametric amplifiers operating in the frequency region mentioned above.

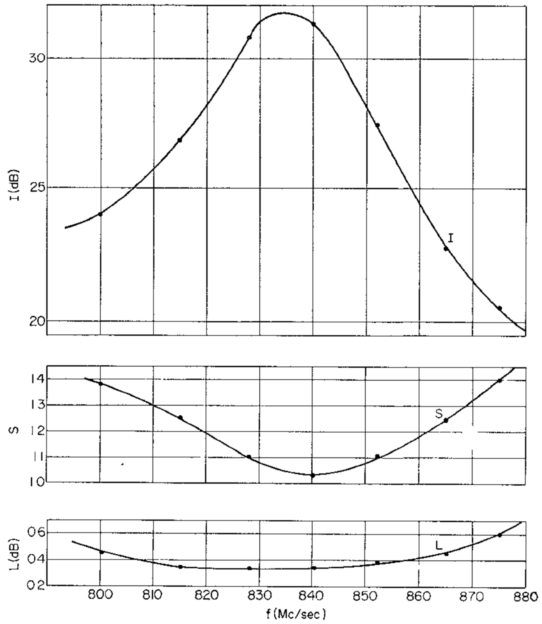

Fig. 4—Performance vs frequency for a fixed polarizing field in the region of 830 mc.

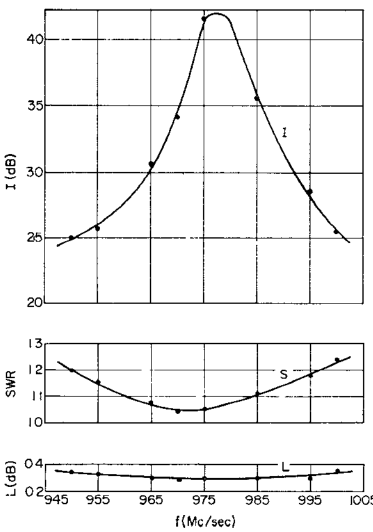

Fig. 5—Performance vs frequency for a fixed polarizing field in the region of 975 mc.

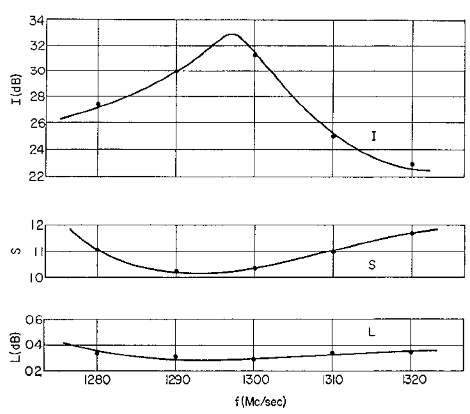

Fig. 6—Performance vs frequency for a fixed polarizing field in the region of 1300 mc.

Appendix

The Behavior of an Imperfect Y-Junction Circulator

In Section II, it was mentioned that when one introduces inside a lossless microwave Y-junction a symmetrically-located piece of ferrite, magnetized along the axis of the junction, one obtains a new nonreciprocal junction with the following characteristics:

1) The scattering matrix $\mathbf{S}^{(f)}$ of the nonreciprocal junction is given in the most general case by (13).
2) The eigenvectors (8) of the scattering matrix $\mathbf{S}^{(r)}$ of the reciprocal junction are still eigenvectors of $\mathbf{S}^{(f)}$.
3) The reciprocity degeneracy of the eigenvalues $s_2^{(r)}$ and $s_3^{(r)}$ is removed and $\mathbf{S}^{(f)}$ has three different eigenvalues $s_i^{(f)} = |s_i| e^{j\phi_i}$ $(i=1, 2, 3)$, ϕ_i being a real number.
4) The elements of the scattering matrix $\mathbf{S}^{(f)}$ are given, as functions of the eigenvalues $s_1^{(f)}$, $s_2^{(f)}$, $s_3^{(f)}$, by the formulas (16)–(18).

One can now have three different situations.

a) The junction containing ferrite is lossless, the $|s_i|$ are equal to unity, and the unit vectors representing the eigenvalues on the complex plane are equally spaced at angles of 120°

$$\phi_2 - \phi_3 = \phi_3 - \phi_1 = \frac{2\pi}{3} \cdot \tag{21}$$

b) The junction has losses, but the conditions (21) are fulfilled.
c) The junction has losses and the conditions (21) are not fulfilled.

In this last case, we will call the junction an "imperfect" Y-junction circulator.

If the dimensions of the ferrite element and of the value of the polarizing magnetic field are not optimal, one does not satisfy conditions (21). In this case, one can write:

$$\phi_2 - \phi_1 = \frac{4\pi}{3} + \delta_2$$
$$\phi_3 - \phi_1 = \frac{2\pi}{3} + \delta_3. \tag{22}$$

As far as the influence of the losses is concerned, we can observe that the eigenvalues s_i $(i=1, 2, 3)$ represent reflection coefficients and, therefore, in the absence of losses $|s_i| = 1$. In the presence of losses, however, one will have $|s_i^{(f)}| \leqq 1$, and we can take this fact into account if we represent the eigenvalues of $\mathbf{S}^{(f)}$ as exponential functions having complex exponents. That is, in the case of an imperfect Y-junction circulator, the most general expression of the generic eigenvalue will be:

$$s_i = e^{(-\alpha_i + j\phi_i)} \quad (i = 1, 2, 3). \tag{23}$$

In particular, if, for a suitable choice of the reference planes of the junction, we assume $\phi_1=0$, we can write the set of eigenvalues as follows:

$$\left.\begin{aligned} s_1 &= e^{-\alpha_1} \\ s_2 &= \exp\left[-\alpha_2 + j\left(\frac{-2\pi}{3} + \delta_2\right)\right] \\ s_3 &= \exp\left[-\alpha_3 + j\left(\frac{2\pi}{3} + \delta_3\right)\right] \end{aligned}\right\} \alpha_i \geqq 0 \ (i = 1, 2, 3). \quad (24)$$

In this description of the imperfect circulator, no element has been included that could alter the symmetry of the junction and therefore all the properties enumerated earlier in points 1)–4) still apply. In particular, the property expressed in point 4) will remain valid. By utilizing the relationships (16)–(18) when the eigenvalues are given by (24), one obtains for the elements of $S^{(f)}$ the following relationships:

$$S' = \frac{1}{3}\left\{e^{-\alpha_1} - \frac{1}{2}\left(e^{-\alpha_3+j\delta_3} + e^{-\alpha_2+j\delta_2}\right) + j\frac{\sqrt{3}}{2}\left(e^{-\alpha_3+j\delta_3} - e^{-\alpha_2+j\delta_2}\right)\right\}$$

$$S'' = \frac{1}{3}\left\{e^{-\alpha_1} + e^{-\alpha_2+j\delta_2} + e^{-\alpha_3+j\delta_3}\right\}$$

$$S''' = \frac{1}{3}\left\{e^{-\alpha_1} - \frac{1}{2}\left(e^{-\alpha_3+j\delta_3} + e^{-\alpha_2+j\delta_2}\right) - j\frac{\sqrt{3}}{2}\left(e^{-\alpha_3+j\delta_3} - e^{-\alpha_2+j\delta_2}\right)\right\}. \quad (25)$$

If the α_i and the δ_i are small, we obtain, by expanding the exponentials in series,

$$S' \cong \frac{1}{3}\left\{\left[-\alpha_1 + \frac{1}{2}(\alpha_3+\alpha_2) - \frac{\sqrt{3}}{2}(\delta_3 - \delta_2)\right] - j\frac{1}{2}\left[\delta_3 + \delta_2 + \sqrt{3}(\alpha_3 - \alpha_2)\right]\right\}$$

$$S'' = \frac{1}{3}\left\{[3 - (\alpha_1+\alpha_2+\alpha_3)] + j[\delta_2+\delta_3]\right\}$$

$$S''' \cong \frac{1}{3}\left\{\left[-\alpha_1 + \frac{1}{2}(\alpha_3+\alpha_2) + \frac{\sqrt{3}}{2}(\delta_3 - \delta_2)\right] - j\frac{1}{2}\left[\delta_3 + \delta_2 - \sqrt{3}(\alpha_3 - \alpha_2)\right]\right\}. \quad (26)$$

By the inspection of (25) and (26), and remembering that $|S'|^2$, $|S''|^2$, and $|S'''|^2$ correspond respectively to the power reflected at the input arm, the power transmitted at the forward arm, the power transmitted at the isolated arm, one derives the following observations: 1) for a lossless circulator, affected by small values of δ_2 and δ_3, $|\delta'|^2 = |S'''|^2$, *i.e.*, the power appearing at the isolated arm equals the power reflected at the input arm; 2) for a lossy circulator, in the general case $|S'|^2 \neq |S'''|^2$ even for small values of the α_i and δ_i ($i=1, 2, 3$) and for suitable combinations of these last quantities, the ratio $|S'|^2/|S'''|^2$ can be quite different from unity.

ACKNOWLEDGMENT

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A Wide-Band UHF Traveling-Wave Variable Reactance Amplifier*

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Summary—**The techniques developed for designing periodically loaded traveling-wave parametric amplifiers using variable-reactance diodes are described in detail. An amplifier was built and tested with two different sets of eight diodes. The performance of the amplifier with each set of diodes agrees substantially with the theoretical predictions, the measured noise figures being about 1.2 db higher than the theoretical values in each case. The gain of the second amplifier varied from a minimum of 6.7 db to more than 13 db over the band from 550 to 930 mc, with a measured noise figure of 2.3 db for wide-band noise inputs in the middle of the band, corresponding to about 4.9 db for single-frequency inputs.**

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GENERAL

TRAVELING-WAVE parametric amplifiers have a number of useful properties, such as wide bandwidth and unilateral amplification, that have been thoroughly discussed in the rapidly expanding litera-